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Rheological Interpretation of Heat Generation Associated with Fatigue of Polycarbonate

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Synopsis

Temperature change was measured of polycarbonate under monotonically increasing tensile and pulsating tensile loads. In the former case, the specimen temperature began to rise when an appreciable amount of viscoelastic strain was noticed on the stress–strain diagram. The rise, θ_V, could be formulated as a function of the viscoelastic strain, ϵ_V:

$$\theta_V = k\epsilon_V^{2/3}.$$

In fatigue tests, the average temperature began to rise immediately after the decrease due to the thermoelastic effect. The amount of the heat generation, α, was nearly constant for each cycle throughout the fatigue process and has a relation to the fatigue life, N_f, $(\alpha - a) \cdot N_f$ = constant, where a is another adjustable constant. From a rheological aspect of dissipation energy, the equation is transformed to a relation between the viscoelastic strain and the fatigue life as $\epsilon_V^{2/3} \cdot N_f$ = constant, which is similar to the one for metals given by Manson and Coffin.[6] The temperature rise in the fatigue was also related to the viscoelastic strain. The relation is of the same form as for static tension but less by a factor of one order.

INTRODUCTION

Although some people explain that fatigue failure of plastics is a result of accelerating rise of temperature in the materials, it is not necessarily the case.[1] Measurements show that under small stress amplitudes the temperature tends to rise to a value only slightly higher than room temperature, and still fatigue fracture occurs. An explanation other than softening due to heat is needed to account for fatigue under such a small stress condition, in some cases of which the heat generated was even associated with an increase, though small, in stiffness of the material.[2] Nevertheless, it must be emphasized that heat generation, however small, must have some relationship to fatigue damage. The authors have tried to interrelate the data presented in their previous paper[3] from a rheological aspect, reserving the possibility for other explanations than softening due to heat generation.

FORMULATION OF MEASUREMENTS

Nonelastic Strain and Temperature Rise

Figure 1 shows stress and temperature change of a polycarbonate specimen being elongated at a constant moderate rate. The temperature, after

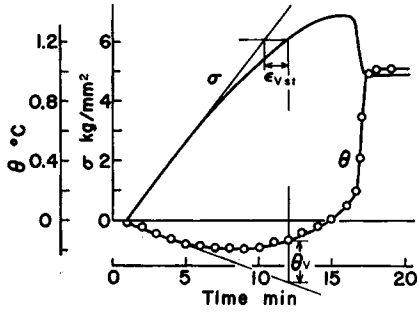

Fig. 1. Changes in stress and temperature of polycarbonate under constant strain rate extension, $\dot{\epsilon} = 0.0042\ \text{min}^{-1}$.

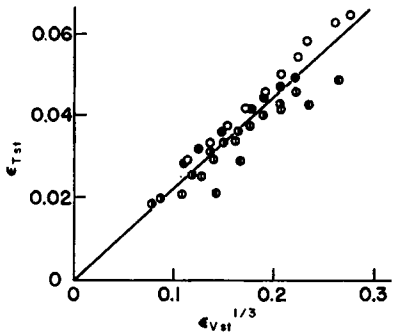

Fig. 2. Relation between total and viscoelastic strains in static tension (eq. (2)): (○) specimen No. 1; (◔) No. 2; (◑) No. 3; (◐) No. 4; (●) No. 5.

a drop due to a thermoelastic effect,[4] tends to increase; at the same time, the viscoelastic strain becomes noticeable on the stress–strain diagram.

The total strain could be separated as follows:

$$\epsilon_{Tst} = \epsilon_{Est} + \epsilon_{Vst} \tag{1}$$

at each stress level. The first component is the part demarcated by the elastic modulus line drawn tangentially to the stress–strain diagram on the earlier part and considered to be elastic; the other component is the difference between the measured total strain and the elastic component and is taken to be viscoelastic. As shown in Figure 2, the total and the viscoelastic strains are admittedly in a relation

$$\epsilon_{Tst} = M_{st}\cdot\epsilon_{Vst}^{1/3},\ M_{st} = 0.225. \tag{2}$$

Likewise, the temperature change due to the viscoelastic straining, θ_V, can be separated from the one due to the thermoelastic effect, $-\theta_E$, when we neglect the heat loss by conduction and radiation and the change in $c\rho$:

$$\theta = -\theta_E + \theta_V. \tag{3}$$

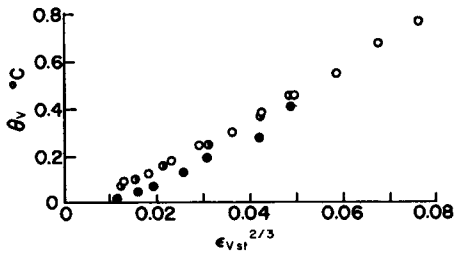

Fig. 3. Temperature rise due to viscoelastic strain in static tension (eq. (4)): (○) specimen No. 1; (◑) No. 3; (●) No. 5.

θ_E was determined from the line tangent to the earliest part of the curve representing the measured temperature change. Then θ_V was graphically obtained as indicated in the figure and found approximately expressible in terms of ϵ_{Vst} as follows:

$$\theta_V = k_{st}\epsilon_{Vst}^{2/3}, \; k_{st} = 8.8°\text{C}. \tag{4}$$

The equation has been found also applicable to material yielding. As already known, the material yields at a tensile strain of about 0.7, at which the temperature rise is nearly 7°C as calculated from the above equation; this almost equals the rise measured of the material yielding under an impulsive tensile load (if the load rate is low; otherwise, the temperature decrease due to heat conduction and radiation cannot be neglected because of a higher temperature rise compared with the rise of as little as 0.1°–0.3°C in the previous case).

Heat Generation Related to Fatigue Life

Under fluctuating tensile loads (minimum stress about 10^8 dyn/cm^2 tensile), the specimen temperature changes as shown in Figure 4. Its initial drop and the following regular fluctuations are caused by a thermoelastic effect of the nonzero mean stress and load fluctuation. The average temperature, on the other hand, is raised monotonically. A simple formulation such as

$$d\theta = \alpha dN - \beta\theta dt \tag{5}$$

is well fitted to the change,[3] where θ is the specimen temperature above room temperature; $d\theta$, dN, and dt are the increments of the temperature, the number of cycles, and the time, respectively; and α and β are heat generation and dissipation factors, respectively. These factors have been evaluated by curve fitting for various stress values. Because the specimens used were identical in form, the β values did not change much, as was expected. The α values were dependent on N_f according to the following relation:

$$(\alpha - a)\, N_f = 7°\text{C}. \tag{6}$$

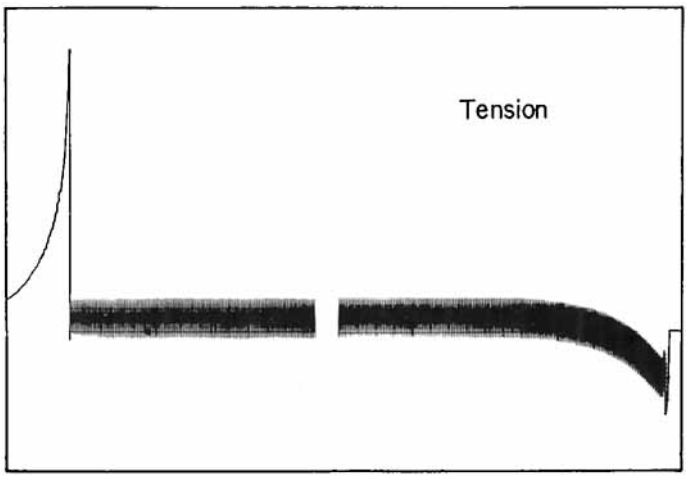

Fig. 4. Temperature change of polycarbonate under pulsating tensile load (reproduced from Higuchi et al.[3]). Record starts at the right.

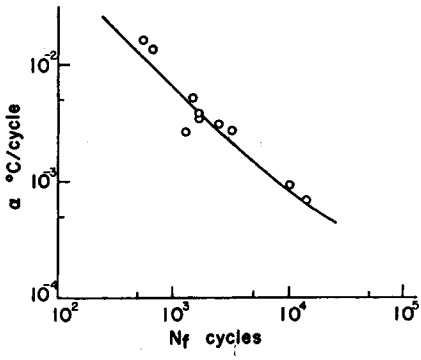

Fig. 5. Relation between heat generation factor α and fatigue life N_f. Line represents eq. (6).

The value of the right-hand side was nearly constant, independently of the stress amplitudes and mean stresses.[3] Here, a, which was introduced for a better fitting, is 1.43×10^{-4}°C/cycle and may be neglected for a rough approximation (Fig. 5).

Figure 6 shows a particular measurement of α, the values of which were obtained with the same specimen but on reloadings after stoppings at 4600 and 7500 cycles in the course of fatigue and after subsequent sufficiently long cooling times. These α values were compared with those for the first loading. They did not appear to differ significantly from one another.

DISCUSSION

Equation (6) suggests that fatigue failure takes place when α has increased to a critical value $U^*/c\rho$; the value of U^* obtained is about 10^8

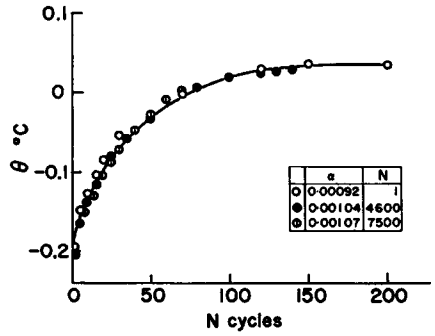

Fig. 6. Average temperature change showing nearly unvaried value of heat generation factor α throughout the fatigue process.

erg/cm³ from the value of $U^*/c\rho = 7°C$ with $c\rho = 0.34$ cal/°C cm³. The α is, on the other hand, associated with the hysteresis loss suffered under cyclic loading. According to the theory of linear viscoelasticity,[5] the dissipation energy for one cycle of a sinusoidal strain, $\epsilon = \epsilon_T \sin \omega t$, is

$$Q = \int_0^{2\pi/\omega} \sigma \dot{\epsilon} dt = \pi E'' \epsilon_T^2, \tag{7}$$

where the additional notations used are current usage. For the present analysis, it is necessary to calculate the dissipation for a triangular fluctuation of the strain, which is represented by a Fourier series:

$$\epsilon = \epsilon_T \frac{8}{\pi^2} \sum_1^\infty (-1)^{n-1} \frac{\sin (2n-1)\omega t}{(2n-1)^2}. \tag{8}$$

Calculating eq. (7) for each term of eq. (8) and adding up all the quantities, we get

$$Q = K\epsilon_T^2 E''(\omega), \; K = \left(\frac{4}{\pi}\right)^3 \sum_1^\infty \frac{1}{(2n-1)^3} \cdot \frac{E''([2n-1]\omega)}{E''(\omega)} \tag{9}$$

and with this,

$$c\rho\alpha = K\epsilon_T^2 E'', \tag{10}$$

where the value of the first term of the series in eq. (9), ~0.26, will be used for an approximate value of K, because the coefficients $1/[(2n-1)^3]$ rapidly decrease in value with increase in n, and $E''([2n-1]\omega)$ in the range of small n values supposedly does not differ much from $E''(\omega)$.

Now let the relationship between the strain amplitudes ϵ_T and ϵ_V under cyclic loadings,

$$\epsilon_T = M\epsilon_V^{1/3}, \tag{11}$$

be similar to that in the case of monotonic elongation. Then eq. (10) is rewritten as follows:

$$\alpha = (KM^2E''/c\rho)\cdot\epsilon_V^{2/3} \tag{12}$$

and hence eq. 6 becomes

$$\epsilon_V^{2/3} \cdot N_f = (U^*/KM^2E'') + (c\rho a/KM^2E'') \cdot N_f \tag{13}$$

or, by ignoring a,

$$\epsilon_V^{2/3} \cdot N_f = (U^*/KM^2E''). \tag{14}$$

Previously one of the authors obtained an experimental relationship.[2]

$$\gamma_V \cdot N_f^{3/2} = \text{const.}$$

between the fatigue life and the viscoelastic shearing strain amplitude, γ_V, in the range of the tension-type fatigue fracture under cyclic torsional loads, which, including the magnitude of the exponent, is similar to eq. (14). Although the relationships are also similar to the Manson-Coffin relationship,[6] the exponent in eq. (14) is viscoelastically determined and differs considerably in magnitude from theirs. We think, nevertheless, that a similar physical meaning may possibly be involved in the Manson-Coffin formula.

The term a is referable to an ineffective component of the work done, acting to heal fatigue damage in one way or another.

The notion that α accumulates linearly is plausible so far as it does not vary in the course of fatigue. It is unlikely, however, that it does not change after a fatigue crack has started to propagate. Accordingly, the number of cycles for a propagating crack to appear is to be taken in place of N_f. The reason for the relation (6) holding well nevertheless may be that N_f, according to our measurement,[7] which differ from the result presented by Manson,[8] is roughly proportional to the life of the crack.

The left-hand side of eq. (14), calculated from the result of our experiment, was

$$U^*/KM^2E'' \cong 10.8. \tag{15}$$

By using $U^* = 10^8$ erg/cm^3 and $K = 2.96$, which were given above, and assuming $M = 0.225$, which is the same as under monotonic load, we get

$$E'' \cong 0.89 \times 10^8 \text{ dyn/cm}^2, \tag{16}$$

which is a value near those measured directly; for instance, a torsional vibration method gave a value[9] of the imaginary component of the shear modulus

$$G'' = 7.87 \times 10^7 \text{ dyn/cm}^2,$$

from which we get

$$E'' = (2.01 \sim 2.24) \times 10^8 \text{ dyn/cm}^2$$

by use of a Poisson's ratio of 0.35 ± 0.07,[10] being assumed real; this is the result of a measurement made at a frequency one order higher than our measurement.

Finally, it is noticed that relation (12) is interestingly the same as that of eq. (4), showing that the heat generation is proportional to $\epsilon_V^{2/3}$; but the factor $KM^2E''/c\rho = k$, say, is found to be about one order less than k_{st}. Therefore

$$\alpha = k\epsilon_V^{2/3}, \; k = 0.66°\text{C/cycle} \tag{17}$$

As is well known, the estimates of fracture surface energy for plastics are in the order of 10^5 erg/cm^2, and the depth of the fracture surface layer is about 0.5×10^{-4} cm on each side.[11,12] Therefore, the fracture energy per unit volume amounts to 10^9 erg/cm^3. If the same energy should be required for fatigue fracture, U^* would be also 10^9 erg/cm^3. The fact is, however, that U^* is 10^8 erg/cm^3, and subsequently the factor k also is one order less than k_{st}. Though not evident, an interpretation is possible that the fracture energy may be of the same order for both the cases, and the volume fraction damaged by fatigue may be about one tenth or less of the volume of the material. The fraction is distributed in such a way that fragments of cell walls, are formed rather irregularly, the damage concentrating there.[3]

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